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Pressure Gradient Influence on MHD Flow for Generalized Burgers' Fluid with Slip Condition

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Abstract
This paper presents a research for magnetohydrodynamic (MHD) flow of an incompressible generalized Burgers' fluid including by an accelerating plate and flowing under the action of pressure gradient. Where the no – slip assumption between the wall and the fluid is no longer valid. The fractional calculus approach is introduced to establish the constitutive relationship of the generalized Burgers' fluid. By using the discrete Laplace transform of the sequential fractional derivatives, a closed form solutions for the velocity and shear stress are obtained in terms of Fox H- function for the following two problems: (i) flow due to a constant pressure gradient, and (ii) flow due to due to a sinusoidal pressure gradient. The solutions for no – slip condition and no magnetic field, can be derived as special cases of our solutions. Furthermore, the effects of various parameters on the velocity distribution characteristics are analyzed and discussed in detail. Comparison between the two cases is also made.

Keywords: Generalized Burgers' fluid, Constant pressure gradient, Sinusoidal pressure gradient, Fox H-function.

I. Introduction

In recent years, the flow of non- Newton fluid has received much attention for their increasing industrial and technological applications, such as extrusion of polymer fluids, exotic lubricants, colloidal and suspension solutions food stuffs and many others. Because of the complicated behavior, there is no model which can alone describe the behavior of all non- Newtonian fluids. For this reason, several constitutive equations for all non-Newtonian fluid models have been proposed. Among them, rate type models have special importance and many researchers are using equations of motion of Maxwell and Oldroyd fluid flow [5, 9, 13, 16, 17]. Recently, a thermodynamic framework has been put into place to develop a rate type model known as Burgers' model [8] which is used to describe the motion of the earth's mantle. The Burgers' model is the preferred model to describe the response of asphalt and asphalt concrete [5]. Many applications of this type of fluid can be found in [3, 6, 11, 14, 15, 18]. Fluids exhibiting boundary slip are important in technological applications, such as the polishing of artificial heart values, polymer melts often exhibit macroscopic wall slip that in general is governed by a non- linear and non- monotone relation between the slip velocity and the traction [4]. Ebaid [1] and Ali [10] studied the effect of magnetic field and slip condition on peristaltic transform. Khaleda [2] gives the exact solution for the slip effect on Stokes and Couette flows due to an oscillating wall. Liancun eta. [19] investigated effect of slip condition on MHD flow of a generalized Oldroyd- B fluid with fractional derivative. They used the fractional approach to write down the constitutive equations for a viscoelastic fluid. A closed form of the velocity distribution and shear stress are obtained in terms of Fox H- function by using the discrete Laplace transform of the sequential fractional derivative.

No attempt has been made regarding the exact solutions for flows due to constant and sinusoidal pressure gradient of generalized Burgers' fluid with fractional derivative and the non- slip condition is no longer valid. The exact solutions for velocity field and shear stress are obtained by using discrete Laplace transform and they are written in term of Fox H- function. Many cases are recovered from our solutions.

II. Governing Equations

The constitutive equations for an incompressible fractional Burger's fluid given by

$$\mathbf{T} = -p\mathbf{I} + \mathbf{S}, \qquad (1+\lambda_1^{\alpha}\tilde{D}_t^{\alpha} + \lambda_2^{\alpha}\tilde{D}_t^{2\alpha})\mathbf{S} = \mu(1+\lambda_3^{\beta}\tilde{D}_t^{\beta})\mathbf{A} \tag{1}$$

where $\mathbf{T}$ denoted the cauchy stress, $-p\mathbf{I}$ is the indeterminate spherical stress, $\mathbf{S}$ is the extra stress tensor, $\mathbf{A} = \mathbf{L} + \mathbf{L}^{T}$ is the first Rivlin- Ericksen tensor with the velocity gradient where $\mathbf{L} = \text{grad}\,\mathbf{V}$, μ is the dynamic viscosity of the fluid, λ_1 and λ_3 $(<\lambda_1)$ are the relaxation and retardation times, respectively, λ_2 is the new material parameter of Burger's fluid, α and β the fractional calculus parameters such that $0 \le \alpha \le \beta \le 1$ and $\tilde{D}_t^p$ the upper convected fractional derivative define by

$$\tilde{D}_t^{\alpha}\mathbf{S} = D_t^{\alpha}\mathbf{S} + (\mathbf{V}.\nabla)\mathbf{S} - \mathbf{LS} - \mathbf{SL}^{\mathrm{T}} \tag{2}$$

$$\tilde{D}_t^{\beta}\mathbf{A} = D_t^{\beta}\mathbf{A} + (\mathbf{V}.\nabla)\mathbf{A} - \mathbf{LA} - \mathbf{AL}^{\mathrm{T}} \tag{3}$$

in which D_t^{α} and D_t^{β} are the fractional differentiation operators of order α and β based on the Riemann-Liouville definition, defined as

$$D_t^p[f(t)] = \frac{1}{\Gamma(1-p)}\frac{d}{dt}\int_0^t \frac{f(\tau)}{(t-\tau)^p}d\tau \qquad ,0 \le p \le 1 \tag{4}$$

here $\Gamma(.)$ denotes the Gamma function and

$$D_t^{2p}\mathbf{S} = D_t^p(D_t^p\mathbf{S}) \tag{5}$$

The model reduced to the generalized Oldroyd- B model when $\lambda_2 = 0$ and if, in addition to that, $\alpha = \beta = 1$ the ordinary Oldroyd- B model will be obtained.

We consider the MHD flow of an incompressible generalized Burger's fluid due to an infinite accelerating plate. For unidirectional flow, we assume that the velocity field and shear stress of the form

$$\mathbf{V} = u(y,t)\mathbf{i} \ \ ,\mathbf{S} = \mathbf{S}(y,t) \tag{6}$$

where u is the velocity and $\mathbf{i}$ is the unit vector in the x- direction .Substituting equation (6) into (1) and taking account of the initial condition

$$\mathbf{S}(y,0) = 0 \quad , \quad y > 0 \tag{7}$$

we obtain

$$(1 + \lambda_1^{\alpha}\mathrm{D}_{\mathrm{t}}^{\alpha} + \lambda_2^{\alpha}\mathrm{D}_{\mathrm{t}}^{2\alpha})\mathrm{S}_{\mathrm{xy}} = \mu(1 + \lambda_3^{\beta}\mathrm{D}_{\mathrm{t}}^{\beta})\partial_{\mathrm{y}}u(y,t) \tag{8}$$

where $\mathrm{S}_{\mathrm{xz}} = \mathrm{S}_{\mathrm{yy}} = \mathrm{S}_{\mathrm{yz}} = \mathrm{S}_{\mathrm{zz}} = 0$, $\mathrm{S}_{\mathrm{xy}} = \mathrm{S}_{\mathrm{yx}}$. Furthermore, it assumes that the conducting fluid is permeated by an imposed magnetic field $\mathbf{B} = [0, \mathrm{B}_0, 0]$ which acts in the positive y- direction. In the low- magnetic Reynolds number approximation, the magnetic body force is represented as $\sigma \mathrm{B}_0^2 u$, where σ is the electrical conductivity of the fluid. Then in the present of a pressure gradient in the x- direction, the equation of motion yields the following scalar equation:

$$\rho\frac{du}{dt} = -\frac{\partial \mathrm{p}}{\partial x} + \mu\frac{\partial}{\partial y}\mathrm{S}_{xy} - \sigma \mathrm{B}_0^2 u \tag{9}$$

where ρ is the constant density of the fluid. Eliminating S_{xy} between Eqs. (8) and (9), we obtain the following fractional differential equation

$$(1 + \lambda_1^{\alpha}\mathrm{D}_{\mathrm{t}}^{\alpha} + \lambda_2^{\alpha}\mathrm{D}_{\mathrm{t}}^{2\alpha})\frac{\partial u}{\partial \mathrm{t}} = -\frac{1}{\rho}(1 + \lambda_1^{\alpha}\mathrm{D}_{\mathrm{t}}^{\alpha} + \lambda_2^{\alpha}\mathrm{D}_{\mathrm{t}}^{2\alpha})\frac{\mathrm{dp}}{\mathrm{dx}} + v(1 + \lambda_3^{\beta}\mathrm{D}_{\mathrm{t}}^{\beta})\frac{\partial^2 u}{\partial y^2} - \mathrm{M}(1 + \lambda_1^{\alpha}\mathrm{D}_{\mathrm{t}}^{\alpha} + \lambda_2^{\alpha}\mathrm{D}_{\mathrm{t}}^{2\alpha})u \tag{10}$$

where $v = \dfrac{\mu}{\rho}$ is the kinematic viscosity and $\mathbf{M} = \dfrac{\sigma \mathrm{B}_0^2}{\rho}$ is the magnetic dimensionless number.

III. Flow induced by a constant pressure gradient:

Let us consider the flow problem of an incompressible generalized Burgers' fluid over an infinite plate at $y \ge 0$ with fluid occupies the space $y > 0$ and flowing under the action of a constant pressure gradient. Also, we assumed the existence of slip boundary between the velocity of fluid at the wall $u(0,t)$ and the speed of the wall, the relative velocity between $u(0,t)$ and the wall is assumed to be proportional to the shear rate at wall. Initially, the system is at rest and at time $t = 0^+$ the fluid is suddenly set in motion due to a constant pressure gradient and by the existence of the slip boundary condition. Referring to Eq. (10), the corresponding fractional partial differential equation that described such flow takes the form

$$\begin{aligned}(1 + \lambda_1^{\alpha}\mathrm{D}_{\mathrm{t}}^{\alpha} + \lambda_2^{\alpha}\mathrm{D}_{\mathrm{t}}^{2\alpha})\frac{\partial u}{\partial \mathrm{t}} = -\mathrm{A}\left(1 + \lambda_1^{\alpha}\frac{t^{-\alpha}}{\Gamma(1-\alpha)} + \lambda_2^{\alpha}\frac{t^{-2\alpha}}{\Gamma(1-2\alpha)}\right) + \\ v(1 + \lambda_3^{\beta}\mathrm{D}_{\mathrm{t}}^{\beta})\frac{\partial^2 u}{\partial t^2} - \mathrm{M}(1 + \lambda_1^{\alpha}\mathrm{D}_{\mathrm{t}}^{\alpha} + \lambda_2^{\alpha}\mathrm{D}_{\mathrm{t}}^{2\alpha})u\end{aligned} \tag{11}$$

where $\mathrm{A} = \frac{1}{\rho}\frac{dp}{dx}$ is the constant pressure gradient

The associated initial and boundary conditions are as follow

$$u(y,0) = \frac{\partial}{\partial t}u(y,0) = 0 \qquad , y > 0 \tag{12}$$

$$u(0,t) = a\,t^b + \theta\frac{\partial u}{\partial y} \qquad , t > 0 \tag{13}$$

$$u(y,t)\,,\,\frac{\partial}{\partial y}u(y,t) \to 0 \quad as\ \ y \to \infty \quad , t > 0 \tag{14}$$

where a and b are constants, θ is the slip strength or slip coefficient. If $\theta = 0$ then the general assumed no-slip boundary condition is obtained. If θ is finite, fluid slip occurs at the wall but its effect depends upon the length scale of the flow.

Employing the non- dimensionless quantities

$$u^* = \frac{u}{(a v^b)^{\frac{1}{2b+1}}} \ ,\ \ y^* = \frac{(a v^b)^{\frac{1}{2b+1}} y}{v} \ ,\ \ t^* = \frac{(a v^b)^{\frac{2}{2b+1}} t}{v} \ ,\ \ \lambda_1^* = \lambda_1\left(\frac{(a v^b)^{\frac{2}{2b+1}}}{v}\right)^{\alpha}$$

$$\lambda_2^* = \lambda_2\left(\frac{(a v^b)^{\frac{2}{2b+1}}}{v}\right)^{2\alpha} \ ,\ \ \lambda_3^* = \lambda_3\left(\frac{(a v^b)^{\frac{2}{2b+1}}}{v}\right)^{\beta} \ ,\ \ \mathrm{M}^* = \frac{\mathrm{M}v}{(a v^b)^{\frac{2}{2b+1}}} \ ,\ \ \theta^* = \frac{(a v^b)^{\frac{2}{2b+1}}\theta}{v}$$

Eqs. (11- 14) in dimensionless form are:

$$(1+\lambda_1^\alpha \mathrm{D}_t^\alpha + \lambda_2^\alpha \mathrm{D}_t^{2\alpha})\frac{\partial u}{\partial t} = -\mathrm{A}\left(1+\lambda_1^\alpha\frac{t^{-\alpha}}{\Gamma(1-\alpha)} + \lambda_2^\alpha\frac{t^{-2\alpha}}{\Gamma(1-2\alpha)}\right) + (1+\lambda_3^\beta \mathrm{D}_t^\beta)\frac{\partial^2 u}{\partial y^2} - \mathrm{M}(1+\lambda_1^\alpha \mathrm{D}_t^\alpha + \lambda_2^\alpha \mathrm{D}_t^{2\alpha})u \tag{15}$$

$$u(y,0) = \frac{\partial}{\partial t}u(y,0) = 0 \qquad , y > 0 \tag{16}$$

$$u(0,t) = t^b + \theta\frac{\partial u}{\partial y} \qquad , t > 0 \tag{17}$$

$$u(y,t)\,,\,\frac{\partial}{\partial y}u(y,t) \to 0 \quad as\ \ y \to \infty \quad , t > 0 \tag{18}$$

where the dimensionless mark "$*$" has been omitted for simplicity.

Now applying Laplace transform principle [7] to Eq. (15) and taking into account the boundary condition (16), we find that

$$\frac{d^2\bar{u}}{dy^2} - \frac{(s+\mathrm{M})(1+\lambda_1^\alpha s^\alpha + \lambda_2^\alpha s^{2\alpha})}{(1+\lambda_3^\beta s^\beta)}\bar{u} = \frac{\mathrm{A}}{s}(1+\lambda_1^\alpha s^\alpha + \lambda_2^\alpha s^{2\alpha}) \tag{19}$$

Subject to boundary conditions

$$\bar{u}(0,s) = \frac{\Gamma(b+1)}{s^{b+1}} + \theta\left.\frac{d\bar{u}}{dy}\right|_{y=0} \tag{20}$$

$$\bar{u}(y,s)\,,\,\frac{\partial}{\partial y}(y,s) \to 0 \quad as\ \ y \to \infty \tag{21}$$

where $\bar{u}(y,s)$ is the image function of $u(y,t)$ and s is a transform parameter. Solving Eqs. (19)- (21), we get

$$\bar{u}(y,s) = \frac{\Gamma(b+1)}{s^{b+1}}\frac{e^{-cy}}{(1+c\theta)} + \frac{\mathrm{A}}{s(s+\mathrm{M})}\frac{e^{-cy}}{(1+c\theta)} - \frac{\mathrm{A}}{s(s+\mathrm{M})} \tag{22}$$

where $c = \left(\frac{(s+\mathrm{M})(1+\lambda_1^\alpha s^\alpha + \lambda_2^\alpha s^{2\alpha})}{(1+\lambda_3^\beta s^\beta)}\right)^{\frac{1}{2}}$

The shear stress can be calculated from Eq. (8), taking Laplace transform of Eq. (8) and introducing Eq. (22), we get

$$\bar{\tau}(y,s) = -(s+\mathrm{M})^{\frac{1}{2}}\left[\frac{(1+\lambda_1^\alpha s^\alpha+\lambda_2^\alpha s^{2\alpha})}{(1+\lambda_3^\beta s^\beta)}\right]^{-\frac{1}{2}}\frac{e^{-ay}}{(1+c\theta)}\left(\frac{\Gamma(b+1)}{s^{b+1}}+\frac{\mathrm{A}}{s(s+\mathrm{M})}\right) \tag{23}$$

where $\bar{\tau}(y,s)$ is the Laplace transform of $\tau(y,t)=\dfrac{\mathrm{S}_{xy}}{\rho(av^b)^{\frac{2}{2b+1}}}$.

In order to avoid the burdensome calculations of residues and contour integrals, we will apply the discrete inverse Laplace transform to get the velocity and the shear stress fields. Now, writing Eq. (22) in series form as

$$\bar{u}(y,s)=\frac{\Gamma(b+1)}{s^{b+1}}+\Gamma(b+1)\sum_{k=1}^{\infty}(-\theta)^k(\frac{\lambda_1^\alpha}{\lambda_3^\beta})^{\frac{k}{2}}\sum_{l=0}^{\infty}\frac{(-\mathrm{M})^l}{l!}\sum_{m=0}^{\infty}\frac{(-\lambda_1^{-\alpha})^m}{m!}\sum_{p=0}^{m}\frac{(\lambda_2^\alpha)^p m!}{p!(m-p)!}\sum_{n=0}^{\infty}\frac{(-\lambda_3^{-\beta})^n}{n!}$$
$$\frac{\Gamma\left(l-\frac{k}{2}\right)\Gamma\left(m-\frac{k}{2}\right)\Gamma\left(n+\frac{k}{2}\right)}{\Gamma\left(-\frac{k}{2}\right)\Gamma\left(-\frac{k}{2}\right)\Gamma\left(\frac{k}{2}\right)s^{\frac{k}{2}(\beta-\alpha-1)+l+\alpha m+\beta n-2\alpha p+b+1}}+\Gamma(b+1)\sum_{k=0}^{\infty}(-\theta)^k\sum_{j=1}^{\infty}\frac{(-y)^j}{j!}(\frac{\lambda_1^\alpha}{\lambda_3^\beta})^{\frac{k+j}{2}}\sum_{l=0}^{\infty}\frac{(-\mathrm{M})^l}{l!}$$
$$\sum_{m=0}^{\infty}\frac{(-\lambda_1^{-\alpha})^m}{m!}\sum_{p=0}^{m}\frac{(\lambda_2^\alpha)^p m!}{p!(m-p)!}\sum_{n=0}^{\infty}\frac{(-\lambda_3^{-\beta})^n}{n!}\frac{\Gamma\left(l-\frac{k+j}{2}\right)\Gamma\left(m-\frac{k+j}{2}\right)\Gamma\left(n+\frac{k+j}{2}\right)}{\Gamma\left(-\frac{k+j}{2}\right)\Gamma\left(-\frac{k+j}{2}\right)\Gamma\left(\frac{k+j}{2}\right)}$$
$$\frac{1}{s^{\frac{k+j}{2}(\beta-\alpha-1)+l+\alpha m+\beta n-2\alpha p+b+1}}+\mathrm{A}\sum_{k=1}^{\infty}(-\theta)^k(\frac{\lambda_1^\alpha}{\lambda_3^\beta})^{\frac{k}{2}}\sum_{l=0}^{\infty}\frac{(-\mathrm{M})^l}{l!}\sum_{m=0}^{\infty}\frac{(-\lambda_1^{-\alpha})^m}{m!}\sum_{p=0}^{m}\frac{(\lambda_2^\alpha)^p m!}{p!(m-p)!}\sum_{i=0}^{\infty}(-\mathrm{M})^i$$
$$\sum_{n=0}^{\infty}\frac{(-\lambda_3^{-\beta})^n}{n!}\frac{\Gamma\left(l-\frac{k}{2}\right)\Gamma\left(m-\frac{k}{2}\right)\Gamma\left(n+\frac{k}{2}\right)}{\Gamma\left(-\frac{k}{2}\right)\Gamma\left(-\frac{k}{2}\right)\Gamma\left(\frac{k}{2}\right)s^{\frac{k}{2}(\beta-\alpha-1)+l+\alpha m+\beta n-2\alpha p+i+2}}+\mathrm{A}\sum_{k=0}^{\infty}(-\theta)^k\sum_{j=1}^{\infty}\frac{(-y)^j}{j!}(\frac{\lambda_1^\alpha}{\lambda_3^\beta})^{\frac{k+j}{2}}$$
$$\sum_{l=0}^{\infty}\frac{(-\mathrm{M})^l}{l!}\sum_{m=0}^{\infty}\frac{(-\lambda_1^{-\alpha})^m}{m!}\sum_{p=0}^{m}\frac{(\lambda_2^\alpha)^p m!}{p!(m-p)!}\sum_{i=0}^{\infty}(-\mathrm{M})^i\sum_{n=0}^{\infty}\frac{(-\lambda_3^{-\beta})^n}{n!}\frac{\Gamma\left(l-\frac{k+j}{2}\right)\Gamma\left(m-\frac{k+j}{2}\right)}{\Gamma\left(-\frac{k+j}{2}\right)\Gamma\left(-\frac{k+j}{2}\right)} \tag{24}$$
$$\frac{\Gamma\left(n+\frac{k+j}{2}\right)}{\Gamma\left(\frac{k+j}{2}\right)s^{\frac{k+j}{2}(\beta-\alpha-1)+l+\alpha m+\beta n-2\alpha p+i+2}}$$

Applying the discrete inverse Laplace transform to Eq. (24), we get

$$u(y,t)=t^b+\Gamma(b+1)\sum_{k=1}^{\infty}(-\theta)^k(\frac{\lambda_1^\alpha}{\lambda_3^\beta})^{\frac{k}{2}}\sum_{l=0}^{\infty}\frac{(-\mathrm{M})^l}{l!}\sum_{m=0}^{\infty}\frac{(-\lambda_1^{-\alpha})^m}{m!}\sum_{p=0}^{m}\frac{(\lambda_2^\alpha)^p m!}{p!(m-p)!}t^{|(\beta-\alpha-1)\frac{k}{2}+l+\alpha m-2\alpha p+b}$$
$$\sum_{n=0}^{\infty}\frac{(-\lambda_3^{-\beta}t^\beta)^n}{n!}\frac{\Gamma\left(l-\frac{k}{2}\right)\Gamma\left(m-\frac{k}{2}\right)\Gamma\left(n+\frac{k}{2}\right)}{\Gamma\left(-\frac{k}{2}\right)\Gamma\left(-\frac{k}{2}\right)\Gamma\left(\frac{k}{2}\right)\Gamma\left(\frac{k}{2}(\beta-\alpha-1)+l+\alpha m-2\alpha p+\beta n+b+1\right)}$$
$$+\Gamma(b+1)\sum_{k=0}^{\infty}(-\theta)^k\sum_{j=1}^{\infty}\frac{(-y)^j}{j!}(\frac{\lambda_1^\alpha}{\lambda_3^\beta})^{\frac{k+j}{2}}\sum_{l=0}^{\infty}\frac{(-\mathrm{M})^l}{l!}\sum_{m=0}^{\infty}\frac{(-\lambda_1^{-\alpha})^m}{m!}\sum_{p=0}^{m}\frac{(\lambda_2^\alpha)^p m!}{p!(m-p)!}t^{(\beta-\alpha-1)\frac{k+j}{2}+l+\alpha m-2\alpha p+b}$$
$$\sum_{n=0}^{\infty}\frac{(-\lambda_3^{-\beta}t^\beta)^n}{n!}\frac{\Gamma\left(l-\frac{k+j}{2}\right)\Gamma\left(m-\frac{k+j}{2}\right)\Gamma\left(n+\frac{k+j}{2}\right)}{\Gamma\left(-\frac{k+j}{2}\right)\Gamma\left(-\frac{k+j}{2}\right)\Gamma\left(\frac{k+j}{2}\right)\Gamma\left(\frac{k+j}{2}(\beta-\alpha-1)+l+\alpha m-2\alpha p+\beta n+b+1\right)}$$

$$+\mathrm{A}\sum_{k=1}^{\infty}(-\theta)^{k}\left(\frac{\lambda_1^{\alpha}}{\lambda_3^{\beta}}\right)^{\frac{k}{2}}\sum_{l=0}^{\infty}\frac{(-\mathrm{M})^{l}}{l!}\sum_{m=0}^{\infty}\frac{(-\lambda_1^{-\alpha})^{m}}{m!}\sum_{p=0}^{m}\frac{(\lambda_2^{\alpha})^{p}\,m!}{p!(m-p)!}\sum_{i=0}^{\infty}(-\mathrm{M})^{i}\;t^{(\beta-\alpha-1)\frac{k}{2}+l+\alpha m-2\alpha p+i+1}$$

$$\sum_{n=0}^{\infty}\frac{(-\lambda_3^{-\beta}\,t^{\beta})^{n}}{n!}\frac{\Gamma\left(l-\frac{k}{2}\right)\Gamma\left(m-\frac{k}{2}\right)\Gamma\left(n+\frac{k}{2}\right)}{\Gamma\left(-\frac{k}{2}\right)\Gamma\left(-\frac{k}{2}\right)\Gamma\left(\frac{k}{2}\right)\Gamma\left(\frac{k}{2}(\beta-\alpha-1)+l+\alpha m-2\alpha p+\beta n+i+2\right)}+ \tag{25}$$

$$\mathrm{A}\sum_{k=0}^{\infty}(-\theta)^{k}\sum_{j=1}^{\infty}\frac{(-y)^{j}}{j!}\left(\frac{\lambda_1^{\alpha}}{\lambda_3^{\beta}}\right)^{\frac{k+j}{2}}\sum_{l=0}^{\infty}\frac{(-\mathrm{M})^{l}}{l!}\sum_{m=0}^{\infty}\frac{(-\lambda_1^{-\alpha})^{m}}{m!}\sum_{p=0}^{m}\frac{(\lambda_2^{\alpha})^{p}\,m!}{p!(m-p)!}\sum_{i=0}^{\infty}(-\mathrm{M})^{i}\;t^{(\beta-\alpha-1)\frac{k+j}{2}+l+\alpha m-2\alpha p+i+1}$$

$$\sum_{n=0}^{\infty}\frac{(-\lambda_3^{-\beta}\,t^{\beta})^{n}}{n!}\frac{\Gamma\left(l-\frac{k+j}{2}\right)\Gamma\left(m-\frac{k+j}{2}\right)\Gamma\left(n+\frac{k+j}{2}\right)}{\Gamma\left(-\frac{k+j}{2}\right)\Gamma\left(-\frac{k+j}{2}\right)\Gamma\left(\frac{k+j}{2}\right)\Gamma\left(\frac{k+j}{2}(\beta-\alpha-1)+l+\alpha m-2\alpha p+\beta n+i+2\right)}$$

In terms of Fox H- function, Eq. (25) takes the simpler from:

$$u(y,t)=t^{b}+\Gamma(b+1)\sum_{k=1}^{\infty}(-\theta)^{k}\left(\frac{\lambda_1^{\alpha}}{\lambda_3^{\beta}}\right)^{\frac{k}{2}}\sum_{l=0}^{\infty}\frac{(-\mathrm{M})^{l}}{l!}\sum_{m=0}^{\infty}\frac{(-\lambda_1^{-\alpha})^{m}}{m!}\sum_{p=0}^{m}\frac{(\lambda_2^{\alpha})^{p}\,m!}{p!(m-p)!}\,t^{|(\beta-\alpha-1)\frac{k}{2}+l+\alpha m-2\alpha p+b}$$

$$\mathrm{H}_{3,5}^{1,3}\left[\frac{t^{\beta}}{\lambda_3^{\beta}}\left|\begin{matrix}(1-l+\frac{k}{2},0),(1-m+\frac{k}{2},0),(1-\frac{k}{2},1)\\(0,1),(1+\frac{k}{2},0),(1+\frac{k}{2},0),(1-\frac{k}{2},0),((\alpha-\beta+1)\frac{k}{2}-l-\alpha m+2\alpha p-b,\beta)\end{matrix}\right.\right]+\Gamma(b+1)\sum_{k=0}^{\infty}(-\theta)^{k}\sum_{j=1}^{\infty}\frac{(-y)^{j}}{j!}\left(\frac{\lambda_1^{\alpha}}{\lambda_3^{\beta}}\right)^{\frac{k+j}{2}}$$

$$\sum_{l=0}^{\infty}\frac{(-\mathrm{M})^{l}}{l!}\sum_{m=0}^{\infty}\frac{(-\lambda_1^{-\alpha})^{m}}{m!}\sum_{p=0}^{m}\frac{(\lambda_2^{\alpha})^{p}\,m!}{p!(m-p)!}\,t^{(\beta-\alpha-1)\frac{k+j}{2}+l+\alpha m-2\alpha p+b}$$

$$\mathrm{H}_{3,5}^{1,3}\left[\frac{t^{\beta}}{\lambda_3^{\beta}}\left|\begin{matrix}(1-l+\frac{k+j}{2},0),(1-m+\frac{k+j}{2},0),(1-\frac{k+j}{2},1)\\(0,1),(1+\frac{k+j}{2},0),(1+\frac{k+j}{2},0),(1-\frac{k+j}{2},0),((\alpha-\beta+1)\frac{k+j}{2}-l-\alpha m+2\alpha p-b,\beta)\end{matrix}\right.\right]+\mathrm{A}\sum_{k=1}^{\infty}(-\theta)^{k}\left(\frac{\lambda_1^{\alpha}}{\lambda_3^{\beta}}\right)^{\frac{k}{2}}\sum_{l=0}^{\infty}\frac{(-\mathrm{M})^{l}}{l!}\sum_{m=0}^{\infty}\frac{(-\lambda_1^{-\alpha})^{m}}{m!}$$

$$\sum_{p=0}^{m}\frac{(\lambda_2^{\alpha})^{p}\,m!}{p!(m-p)!}\sum_{i=0}^{\infty}(-\mathrm{M})^{i}\;t^{(\beta-\alpha-1)\frac{k}{2}+l+\alpha m-2\alpha p+i+1}\;\mathrm{H}_{3,5}^{1,3}\left[\frac{t^{\beta}}{\lambda_3^{\beta}}\left|\begin{matrix}(1-l+\frac{k}{2},0),(1-m+\frac{k}{2},0),(1-\frac{k}{2},1)\\(0,1),(1+\frac{k}{2},0),(1+\frac{k}{2},0),(1-\frac{k}{2},0),((\alpha-\beta+1)\frac{k}{2}-l-\alpha m+2\alpha p-i-1,\beta)\end{matrix}\right.\right]+$$

$$\mathrm{A}\sum_{k=0}^{\infty}(-\theta)^{k}\sum_{j=1}^{\infty}\frac{(-y)^{j}}{j!}\left(\frac{\lambda_1^{\alpha}}{\lambda_3^{\beta}}\right)^{\frac{k+j}{2}}\sum_{l=0}^{\infty}\frac{(-\mathrm{M})^{l}}{l!}\sum_{m=0}^{\infty}\frac{(-\lambda_1^{-\alpha})^{m}}{m!}\sum_{p=0}^{m}\frac{(\lambda_2^{\alpha})^{p}\,m!}{p!(m-p)!}\sum_{i=0}^{\infty}(-\mathrm{M})^{i}\;t^{(\beta-\alpha-1)\frac{k+j}{2}+l+\alpha m-2\alpha p+i+1}$$

$$\mathrm{H}_{3,5}^{1,3}\left[\frac{t^{\beta}}{\lambda_3^{\beta}}\left|\begin{matrix}(1-l+\frac{k+j}{2},0),(1-m+\frac{k+j}{2},0),(1-\frac{k+j}{2},1)\\(0,1),(1+\frac{k+j}{2},0),(1+\frac{k+j}{2},0),(1-\frac{k+j}{2},0),((\alpha-\beta+1)\frac{k+j}{2}-l-\alpha m+2\alpha p-i-1,\beta)\end{matrix}\right.\right] \tag{26}$$

where the property of the Fox H- function is

$$\sum_{n=0}^{\infty}\frac{(-z)^{n}\prod_{j=1}^{p}\Gamma(a_j+A_j n)}{n!\prod_{j=1}^{q}\Gamma(b_j+B_j n)}=\mathrm{H}_{p,q+1}^{1,p}\left[z\left|\begin{matrix}(1-a_1,A_1),\cdots,(1-a_p,A_p)\\(0,1),(1-b_1,B_1),\cdots,(1-b_q,B_q)\end{matrix}\right.\right]$$

The solution Eq. (26) should satisfy the boundary condition Eq. (17). To see this, from Eq. (26) it is easy to obtain

$$u(0,t)=t^{b}+\Gamma(b+1)\sum_{k=1}^{\infty}(-\theta)^{k}\left(\frac{\lambda_1^{\alpha}}{\lambda_3^{\beta}}\right)^{\frac{k}{2}}\sum_{l=0}^{\infty}\frac{(-\mathrm{M})^{l}}{l!}\sum_{m=0}^{\infty}\frac{(-\lambda_1^{-\alpha})^{m}}{m!}\sum_{p=0}^{m}\frac{(\lambda_2^{\alpha})^{p}m!}{p!(m-p)!}t^{(\beta-\alpha-1)\frac{k}{2}+l+\alpha m-2\alpha p+b}$$
$$\mathrm{H}_{3,5}^{1,3}\left[\frac{t^{\beta}}{\lambda_3^{\beta}}\middle|_{(0,1),(1+\frac{k}{2},0),(1+\frac{k}{2},0),(1-\frac{k}{2},0),((\alpha-\beta+1)\frac{k}{2}-l-\alpha m+2\alpha p-b,\beta)}^{(1-l+\frac{k}{2},0),(1-m+\frac{k}{2},0),(1-\frac{k}{2},1)}\right]+\mathrm{A}\sum_{k=1}^{\infty}(-\theta)^{k}\left(\frac{\lambda_1^{\alpha}}{\lambda_3^{\beta}}\right)^{\frac{k}{2}}\sum_{l=0}^{\infty}\frac{(-\mathrm{M})^{l}}{l!}\sum_{m=0}^{\infty}\frac{(-\lambda_1^{-\alpha})^{m}}{m!}$$
$$\sum_{p=0}^{m}\frac{(\lambda_2^{\alpha})^{p}m!}{p!(m-p)!}\sum_{i=0}^{\infty}(-\mathrm{M})^{i}\,t^{(\beta-\alpha-1)\frac{k}{2}+l+\alpha m-2\alpha p+i+1}\mathrm{H}_{3,5}^{1,3}\left[\frac{t^{\beta}}{\lambda_3^{\beta}}\middle|_{(0,1),(1+\frac{k}{2},0),(1+\frac{k}{2},0),(1-\frac{k}{2},0),((\alpha-\beta+1)\frac{k}{2}-l-\alpha m+2\alpha p-i-1,\beta)}^{(1-l+\frac{k}{2},0),(1-m+\frac{k}{2},0),(1-\frac{k}{2},1)}\right]$$

and

$$\left.\frac{\partial u(y,t)}{\partial y}\right|_{y=0}=-\Gamma(b+1)\sum_{k=0}^{\infty}(-\theta)^{k}\left(\frac{\lambda_1^{\alpha}}{\lambda_3^{\beta}}\right)^{\frac{k+1}{2}}\sum_{l=0}^{\infty}\frac{(-\mathrm{M})^{l}}{l!}\sum_{m=0}^{\infty}\frac{(-\lambda_1^{-\alpha})^{m}}{m!}\sum_{p=0}^{m}\frac{(\lambda_2^{\alpha})^{p}m!}{p!(m-p)!}t^{(\beta-\alpha-1)\frac{k+1}{2}+l+\alpha m-2\alpha p+b}$$
$$\mathrm{H}_{3,5}^{1,3}\left[\frac{t^{\beta}}{\lambda_3^{\beta}}\middle|_{(0,1),(1+\frac{k+1}{2},0),(1+\frac{k+1}{2},0),(1-\frac{k+1}{2},0),((\alpha-\beta+1)\frac{k+1}{2}-l-\alpha m+2\alpha p-b,\beta)}^{(1-l+\frac{k+1}{2},0),(1-m+\frac{k+1}{2},0),(1-\frac{k+1}{2},1)}\right]+\mathrm{A}\sum_{k=1}^{\infty}(-\theta)^{k}\left(\frac{\lambda_1^{\alpha}}{\lambda_3^{\beta}}\right)^{\frac{k+1}{2}}\sum_{l=0}^{\infty}\frac{(-\mathrm{M})^{l}}{l!}\sum_{m=0}^{\infty}\frac{(-\lambda_1^{-\alpha})^{m}}{m!}$$
$$\sum_{p=0}^{m}\frac{(\lambda_2^{\alpha})^{p}m!}{p!(m-p)!}\sum_{i=0}^{\infty}(-\mathrm{M})^{i}\,t^{(\beta-\alpha-1)\frac{k+1}{2}+l+\alpha m-2\alpha p+i+1}\mathrm{H}_{3,5}^{1,3}\left[\frac{t^{\beta}}{\lambda_3^{\beta}}\middle|_{(0,1),(1+\frac{k+1}{2},0),(1+\frac{k+1}{2},0),(1-\frac{k+1}{2},0),((\alpha-\beta+1)\frac{k+1}{2}-l-\alpha m+2\alpha p-i-1,\beta)}^{(1-l+\frac{k+1}{2},0),(1-m+\frac{k+1}{2},0),(1-\frac{k+1}{2},1)}\right]$$

then

$$\left.\frac{\partial u(y,t)}{\partial y}\right|_{y=0}=-\Gamma(b+1)\sum_{k=0}^{\infty}(-\theta)^{k}\left(\frac{\lambda_1^{\alpha}}{\lambda_3^{\beta}}\right)^{\frac{k+1}{2}}\sum_{l=0}^{\infty}\frac{(-\mathrm{M})^{l}}{l!}\sum_{m=0}^{\infty}\frac{(-\lambda_1^{-\alpha})^{m}}{m!}\sum_{p=0}^{m}\frac{(\lambda_2^{\alpha})^{p}m!}{p!(m-p)!}t^{(\beta-\alpha-1)\frac{k+1}{2}+l+\alpha m-2\alpha p+b}$$
$$\mathrm{H}_{3,5}^{1,3}\left[\frac{t^{\beta}}{\lambda_3^{\beta}}\middle|_{(0,1),(1+\frac{k+1}{2},0),(1+\frac{k+1}{2},0),(1-\frac{k+1}{2},0),((\alpha-\beta+1)\frac{k+1}{2}-l-\alpha m+2\alpha p-b,\beta)}^{(1-l+\frac{k+1}{2},0),(1-m+\frac{k+1}{2},0),(1-\frac{k+1}{2},1)}\right]+\mathrm{A}\sum_{k=1}^{\infty}(-\theta)^{k}\left(\frac{\lambda_1^{\alpha}}{\lambda_3^{\beta}}\right)^{\frac{k+1}{2}}\sum_{l=0}^{\infty}\frac{(-\mathrm{M})^{l}}{l!}\sum_{m=0}^{\infty}\frac{(-\lambda_1^{-\alpha})^{m}}{m!}$$
$$\sum_{p=0}^{m}\frac{(\lambda_2^{\alpha})^{p}m!}{p!(m-p)!}\sum_{i=0}^{\infty}(-\mathrm{M})^{i}\,t^{(\beta-\alpha-1)\frac{k+1}{2}+l+\alpha m-2\alpha p+i+1}\mathrm{H}_{3,5}^{1,3}\left[\frac{t^{\beta}}{\lambda_3^{\beta}}\middle|_{(0,1),(1+\frac{k+1}{2},0),(1+\frac{k+1}{2},0),(1-\frac{k+1}{2},0),((\alpha-\beta+1)\frac{k+1}{2}-l-\alpha m+2\alpha p-i-1,\beta)}^{(1-l+\frac{k+1}{2},0),(1-m+\frac{k+1}{2},0),(1-\frac{k+1}{2},1)}\right]$$

i.e. $u(0,t)=t^{b}+\theta\dfrac{\partial u(y,t)}{\partial y}$

Adopting the similar procedure in Eq. (23), we obtain the shear stress:

$$\tau=-\Gamma(b+1)\sum_{k=0}^{\infty}(-\theta)^{k}\sum_{j=0}^{\infty}\frac{(-y)^{j}}{j!}\left(\frac{\lambda_1^{\alpha}}{\lambda_3^{\beta}}\right)^{\frac{k+j-1}{2}}\sum_{l=0}^{\infty}\frac{(-\mathrm{M})^{l}}{l!}\sum_{m=0}^{\infty}\frac{(-\lambda_1^{-\alpha})^{m}}{m!}\sum_{p=0}^{m}\frac{(\lambda_2^{\alpha})^{p}m!}{p!(m-p)!}$$
$$t^{(\beta-\alpha-1)\frac{k+j-1}{2}+l+\alpha m-2\alpha p+b-1}\mathrm{H}_{3,5}^{1,3}\left[\frac{t^{\beta}}{\lambda_3^{\beta}}\middle|_{(0,1),(1+\frac{k+j+1}{2},0),(1+\frac{k+j-1}{2},0),(1-\frac{k+j-1}{2},0),((\alpha-\beta+1)\frac{k+j-1}{2}-l-\alpha m+2\alpha p-b+1,\beta)}^{(1-l+\frac{k+j-1}{2},0),(1-m+\frac{k+j-1}{2},0),(1-\frac{k+j-1}{2},1)}\right]$$
$$-\mathrm{A}\sum_{k=0}^{\infty}(-\theta)^{k}\sum_{j=0}^{\infty}\frac{(-y)^{j}}{j!}\left(\frac{\lambda_1^{\alpha}}{\lambda_3^{\beta}}\right)^{\frac{k+j-1}{2}}\sum_{l=0}^{\infty}\frac{(-\mathrm{M})^{l}}{l!}\sum_{m=0}^{\infty}\frac{(-\lambda_1^{-\alpha})^{m}}{m!}\sum_{p=0}^{m}\frac{(\lambda_2^{\alpha})^{p}m!}{p!(m-p)!}\sum_{i=0}^{\infty}(-\mathrm{M})^{i} \tag{27}$$
$$t^{(\beta-\alpha-1)\frac{k+j-1}{2}+l+\alpha m-2\alpha p+i}\mathrm{H}_{3,5}^{1,3}\left[\frac{t^{\beta}}{\lambda_3^{\beta}}\middle|_{(0,1),(1+\frac{k+j+1}{2},0),(1+\frac{k+j-1}{2},0),(1-\frac{k+j-1}{2},0),((\alpha-\beta+1)\frac{k+j-1}{2}-l-\alpha m+2\alpha p-i,\beta)}^{(1-l+\frac{k+j+1}{2},0),(1-m+\frac{k+j-1}{2},0),(1-\frac{k+j-1}{2},1)}\right]$$

Special Cases:

1- If $\theta=0$ then the no- slip condition is obtained. In this special case Eqs. (26) and (27) are simplified to

$$u(y,t)=t^{b}+\Gamma(b+1)\sum_{j=1}^{\infty}\frac{(-y)^{j}}{j!}\left(\frac{\lambda_1^{\alpha}}{\lambda_3^{\beta}}\right)^{\frac{j}{2}}\sum_{l=0}^{\infty}\frac{(-\mathrm{M})^{l}}{l!}\sum_{m=0}^{\infty}\frac{(-\lambda_1^{-\alpha})^{m}}{m!}\sum_{p=0}^{m}\frac{(\lambda_2^{\alpha})^{p}\,m!}{p!(m-p)!}\,t^{(\beta-\alpha-1)\frac{j}{2}+l+\alpha m-2\alpha p+b}$$

$$\mathrm{H}_{3,5}^{1,3}\left[\frac{t^{\beta}}{\lambda_3^{\beta}}\Bigg|_{(0,1),(1+\frac{j}{2},0),(1+\frac{j}{2},0),(1-\frac{j}{2},0),((\alpha-\beta+1)\frac{j}{2}-l-\alpha m+2\alpha p-b,\beta)}^{(1-l+\frac{j}{2},0),(1-m+\frac{j}{2},0),(1-\frac{j}{2},1)}\right]+\mathrm{A}\sum_{j=1}^{\infty}\frac{(-y)^{j}}{j!}\left(\frac{\lambda_1^{\alpha}}{\lambda_3^{\beta}}\right)^{\frac{j}{2}}\sum_{l=0}^{\infty}\frac{(-\mathrm{M})^{l}}{l!}\sum_{m=0}^{\infty}\frac{(-\lambda_1^{-\alpha})^{m}}{m!}$$

$$\sum_{p=0}^{m}\frac{(\lambda_2^{\alpha})^{p}\,m!}{p!(m-p)!}\sum_{i=0}^{\infty}(-\mathrm{M})^{i}\;t^{(\beta-\alpha-1)\frac{j}{2}+l+\alpha m-2\alpha p+i+1}\;\mathrm{H}_{3,5}^{1,3}\left[\frac{t^{\beta}}{\lambda_3^{\beta}}\Bigg|_{(0,1),(1+\frac{j}{2},0),(1+\frac{j}{2},0),(1-\frac{j}{2},0),((\alpha-\beta+1)\frac{j}{2}-l-\alpha m+2\alpha p-i-1,\beta)}^{(1-l+\frac{j}{2},0),(1-m+\frac{j}{2},0),(1-\frac{j}{2},1)}\right]$$

$$\tau(y,t)=-\Gamma(b+1)\sum_{j=0}^{\infty}\frac{(-y)^{j}}{j!}\left(\frac{\lambda_1^{\alpha}}{\lambda_3^{\beta}}\right)^{\frac{j-1}{2}}\sum_{l=0}^{\infty}\frac{(-\mathrm{M})^{l}}{l!}\sum_{m=0}^{\infty}\frac{(-\lambda_1^{-\alpha})^{m}}{m!}\sum_{p=0}^{m}\frac{(\lambda_2^{\alpha})^{p}\,m!}{p!(m-p)!}$$

$$t^{(\beta-\alpha-1)\frac{j-1}{2}+l+\alpha m-2\alpha p+b-1}\;\mathrm{H}_{3,5}^{1,3}\left[\frac{t^{\beta}}{\lambda_3^{\beta}}\Bigg|_{(0,1),(1+\frac{j+1}{2},0),(1+\frac{j-1}{2},0),(1-\frac{j-1}{2},0),((\alpha-\beta+1)\frac{j-1}{2}-l-\alpha m+2\alpha p-b+1,\beta)}^{(1-l+\frac{j-1}{2},0),(1-m+\frac{j-1}{2},0),(1-\frac{j-1}{2},1)}\right]-$$

$$\mathrm{A}\sum_{j=0}^{\infty}\frac{(-y)^{j}}{j!}\left(\frac{\lambda_1^{\alpha}}{\lambda_3^{\beta}}\right)^{\frac{j-1}{2}}\sum_{l=0}^{\infty}\frac{(-\mathrm{M})^{l}}{l!}\sum_{m=0}^{\infty}\frac{(-\lambda_1^{-\alpha})^{m}}{m!}\sum_{p=0}^{m}\frac{(\lambda_2^{\alpha})^{p}\,m!}{p!(m-p)!}\sum_{i=0}^{\infty}(-\mathrm{M})^{i}$$

$$t^{(\beta-\alpha-1)\frac{j-1}{2}+l+\alpha m-2\alpha p+i}\;\mathrm{H}_{3,5}^{1,3}\left[\frac{t^{\beta}}{\lambda_3^{\beta}}\Bigg|_{(0,1),(1+\frac{j+1}{2},0),(1+\frac{j-1}{2},0),(1-\frac{j-1}{2},0),((\alpha-\beta+1)\frac{j-1}{2}-l-\alpha m+2\alpha p-i,\beta)}^{(1-l+\frac{j+1}{2},0),(1-m+\frac{j-1}{2},0),(1-\frac{j-1}{2},1)}\right]$$

2- If $\theta\neq 0$ and $\mathrm{M}=0$ then Eqs. (26) and (27) reduce to

$$u(y,t)=t^{b}+\Gamma(b+1)\sum_{k=1}^{\infty}(-\theta)^{k}\left(\frac{\lambda_1^{\alpha}}{\lambda_3^{\beta}}\right)^{\frac{k}{2}}\sum_{m=0}^{\infty}\frac{(-\lambda_1^{-\alpha})^{m}}{m!}\sum_{p=0}^{m}\frac{(\lambda_2^{\alpha})^{p}\,m!}{p!(m-p)!}\,t^{|(\beta-\alpha-1)\frac{k}{2}+\alpha m-2\alpha p+b}$$

$$\mathrm{H}_{2,4}^{1,2}\left[\frac{t^{\beta}}{\lambda_3^{\beta}}\Bigg|_{(0,1),(1+\frac{k}{2},0),(1-\frac{k}{2},0),((\alpha-\beta+1)\frac{k}{2}-\alpha m+2\alpha p-b,\beta)}^{(1-m+\frac{k}{2},0),(1-\frac{k}{2},1)}\right]+\Gamma(b+1)\sum_{k=0}^{\infty}(-\theta)^{k}\sum_{j=1}^{\infty}\frac{(-y)^{j}}{j!}\left(\frac{\lambda_1^{\alpha}}{\lambda_3^{\beta}}\right)^{\frac{k+j}{2}}$$

$$\sum_{m=0}^{\infty}\frac{(-\lambda_1^{-\alpha})^{m}}{m!}\sum_{p=0}^{m}\frac{(\lambda_2^{\alpha})^{p}\,m!}{p!(m-p)!}\,t^{(\beta-\alpha-1)\frac{k+j}{2}+\alpha m-2\alpha p+b}\;\mathrm{H}_{2,4}^{1,2}\left[\frac{t^{\beta}}{\lambda_3^{\beta}}\Bigg|_{(0,1),(1+\frac{k+j}{2},0),(1-\frac{k+j}{2},0),((\alpha-\beta+1)\frac{k+j}{2}-\alpha m+2\alpha p-b,\beta)}^{(1-m+\frac{k+j}{2},0),(1-\frac{k+j}{2},1)}\right]$$

$$+\mathrm{A}\sum_{k=1}^{\infty}(-\theta)^{k}\left(\frac{\lambda_1^{\alpha}}{\lambda_3^{\beta}}\right)^{\frac{k}{2}}\sum_{m=0}^{\infty}\frac{(-\lambda_1^{-\alpha})^{m}}{m!}\sum_{p=0}^{m}\frac{(\lambda_2^{\alpha})^{p}\,m!}{p!(m-p)!}\;t^{(\beta-\alpha-1)\frac{k}{2}+\alpha m-2\alpha p+1}$$

$$\mathrm{H}_{2,4}^{1,2}\left[\frac{t^{\beta}}{\lambda_3^{\beta}}\Bigg|_{(0,1),(1+\frac{k}{2},0),(1-\frac{k}{2},0),((\alpha-\beta+1)\frac{k}{2}-\alpha m+2\alpha p-1,\beta)}^{(1-m+\frac{k}{2},0),(1-\frac{k}{2},1)}\right]+\mathrm{A}\sum_{k=0}^{\infty}(-\theta)^{k}\sum_{j=1}^{\infty}\frac{(-y)^{j}}{j!}\left(\frac{\lambda_1^{\alpha}}{\lambda_3^{\beta}}\right)^{\frac{k+j}{2}}\sum_{m=0}^{\infty}\frac{(-\lambda_1^{-\alpha})^{m}}{m!}$$

$$\sum_{p=0}^{m}\frac{(\lambda_2^{\alpha})^{p}\,m!}{p!(m-p)!}\,t^{(\beta-\alpha-1)\frac{k+j}{2}+\alpha m-2\alpha p+1}\;\mathrm{H}_{2,4}^{1,2}\left[\frac{t^{\beta}}{\lambda_3^{\beta}}\Bigg|_{(0,1),(1+\frac{k+j}{2},0),(1-\frac{k+j}{2},0),((\alpha-\beta+1)\frac{k+j}{2}-\alpha m+2\alpha p-1,\beta)}^{(1-m+\frac{k+j}{2},0),(1-\frac{k+j}{2},1)}\right]$$

$$\tau(y,t) = -\Gamma(b+1)\sum_{k=0}^{\infty}(-\theta)^k \sum_{j=0}^{\infty}\frac{(-y)^j}{j!}\left(\frac{\lambda_1^\alpha}{\lambda_3^\beta}\right)^{\frac{k+j-1}{2}}\sum_{m=0}^{\infty}\frac{(-\lambda_1^{-\alpha})^m}{m!}\sum_{p=0}^{m}\frac{(\lambda_2^\alpha)^p m!}{p!(m-p)!}$$
$$t^{(\beta-\alpha-1)\frac{k+j-1}{2}+\alpha m-2\alpha p+b-1}\,\mathrm{H}^{1,2}_{2,4}\left[\frac{t^\beta}{\lambda_3^\beta}\left|\begin{matrix}(1-m+\frac{k+j-1}{2},0),(1-\frac{k+j-1}{2},1)\\(0,1),(1+\frac{k+j-1}{2},0),(1-\frac{k+j-1}{2},0),((\alpha-\beta+1)\frac{k+j-1}{2}-\alpha m+2\alpha p-b+1,\beta)\end{matrix}\right.\right]$$
$$-\mathrm{A}\sum_{k=0}^{\infty}(-\theta)^k \sum_{j=0}^{\infty}\frac{(-y)^j}{j!}\left(\frac{\lambda_1^\alpha}{\lambda_3^\beta}\right)^{\frac{k+j-1}{2}}\sum_{m=0}^{\infty}\frac{(-\lambda_1^{-\alpha})^m}{m!}\sum_{p=0}^{m}\frac{(\lambda_2^\alpha)^p m!}{p!(m-p)!}$$
$$t^{(\beta-\alpha-1)\frac{k+j-1}{2}+\alpha m-2\alpha p}\,\mathrm{H}^{1,2}_{2,4}\left[\frac{t^\beta}{\lambda_3^\beta}\left|\begin{matrix}(1-m+\frac{k+j-1}{2},0),(1-\frac{k+j-1}{2},1)\\(0,1),(1+\frac{k+j-1}{2},0),(1-\frac{k+j-1}{2},0),((\alpha-\beta+1)\frac{k+j-1}{2}-\alpha m+2\alpha p,\beta)\end{matrix}\right.\right]$$

which correspond to the flow of a generalized Burgers' magnetic field effect.

3- Making $\lambda_2 \to 0$ and $\mathrm{A} \to 0$ in Eqs. (26) and (27) the solutions corresponds to slip effects of a generalized Oldroyd- B fluid in absence of pressure gradient can be recovered, as found by Zheng … etc in [19].

4- If we set $\lambda_2 \to 0$, $\theta = 0$ and $\mathrm{M} = 0$ in Eqs. (26) and (27) the similar solutions for generalized Oldroyd- B fluid are recovered, as found by Hyder … etc in [12].

IV. Flow due to a sinusoidal pressure gradient:

Let us consider the flow problem of generalized Burgers' fluid bounded by an infinite plane wall at $y = 0$, under the action of sinusoidal pressure gradient with the same initial and boundary conditions, Eqs. (16- 18).
In this case the governing equation can be written as

$$(1+\lambda_1^\alpha \mathrm{D}_t^\alpha + \lambda_2^\alpha \mathrm{D}_t^{2\alpha})\frac{\partial u}{\partial t} = -p_0(1+\lambda_1^\alpha \mathrm{D}_t^\alpha + \lambda_2^\alpha \mathrm{D}_t^{2\alpha})\cos(wt) + (1+\lambda_3^\beta \mathrm{D}_t^\beta)\frac{\partial^2 u}{\partial y^2} - \mathrm{M}(1+\lambda_1^\alpha \mathrm{D}_t^\alpha + \lambda_2^\alpha \mathrm{D}_t^{2\alpha})u \tag{28}$$

where $\frac{dp}{dx} = \rho\, p_0 \cos(wt) \Rightarrow \frac{1}{\rho}\frac{dp}{dx} = p_0 \cos(wt)$ and p_0 is constant. The associated initial and boundary condition are as given in Eqs. (16- 18).
Again, by similar procedure as in the previous case the velocity field is found in the form of

$$u(y,t) = t^b + \Gamma(b+1)\sum_{k=1}^{\infty}(-\theta)^k\left(\frac{\lambda_1^\alpha}{\lambda_3^\beta}\right)^{\frac{k}{2}}\sum_{l=0}^{\infty}\frac{(-\mathrm{M})^l}{l!}\sum_{m=0}^{\infty}\frac{(-\lambda_1^{-\alpha})^m}{m!}\sum_{p=0}^{m}\frac{(\lambda_2^\alpha)^p m!}{p!(m-p)!}\,t^{|(\beta-\alpha-1)\frac{k}{2}+l+\alpha m-2\alpha p+b}$$
$$\mathrm{H}^{1,3}_{3,5}\left[\frac{t^\beta}{\lambda_3^\beta}\left|\begin{matrix}(1-l+\frac{k}{2},0),(1-m+\frac{k}{2},0),(1-\frac{k}{2},1)\\(0,1),(1+\frac{k}{2},0),(1+\frac{k}{2},0),(1-\frac{k}{2},0),((\alpha-\beta+1)\frac{k}{2}-l-\alpha m+2\alpha p-b,\beta)\end{matrix}\right.\right] + \Gamma(b+1)\sum_{k=0}^{\infty}(-\theta)^k\sum_{j=1}^{\infty}\frac{(-y)^j}{j!}\left(\frac{\lambda_1^\alpha}{\lambda_3^\beta}\right)^{\frac{k+j}{2}}$$
$$\sum_{l=0}^{\infty}\frac{(-\mathrm{M})^l}{l!}\sum_{m=0}^{\infty}\frac{(-\lambda_1^{-\alpha})^m}{m!}\sum_{p=0}^{m}\frac{(\lambda_2^\alpha)^p m!}{p!(m-p)!}\,t^{(\beta-\alpha-1)\frac{k+j}{2}+l+\alpha m-2\alpha p+b}$$
$$\mathrm{H}^{1,3}_{3,5}\left[\frac{t^\beta}{\lambda_3^\beta}\left|\begin{matrix}(1-l+\frac{k+j}{2},0),(1-m+\frac{k+j}{2},0),(1-\frac{k+j}{2},1)\\(0,1),(1+\frac{k+j}{2},0),(1+\frac{k+j}{2},0),(1-\frac{k+j}{2},0),((\alpha-\beta+1)\frac{k+j}{2}-l-\alpha m+2\alpha p-b,\beta)\end{matrix}\right.\right] + p_0\sum_{k=1}^{\infty}(-\theta)^k\left(\frac{\lambda_1^\alpha}{\lambda_3^\beta}\right)^{\frac{k}{2}}\sum_{l=0}^{\infty}\frac{(-\mathrm{M})^l}{l!}$$
$$\sum_{m=0}^{\infty}\frac{(-\lambda_1^{-\alpha})^m}{m!}\sum_{p=0}^{m}\frac{(\lambda_2^\alpha)^p m!}{p!(m-p)!}\sum_{i=0}^{\infty}(-\mathrm{M})^i\sum_{r=0}^{\infty}(-w^2)^r\,t^{(\beta-\alpha-1)\frac{k}{2}+l+\alpha m-2\alpha p+i+2r+1} \tag{29}$$
$$\mathrm{H}^{1,3}_{3,5}\left[\frac{t^\beta}{\lambda_3^\beta}\left|\begin{matrix}(1-l+\frac{k}{2},0),(1-m+\frac{k}{2},0),(1-\frac{k}{2},1)\\(0,1),(1+\frac{k}{2},0),(1+\frac{k}{2},0),(1-\frac{k}{2},0),((\alpha-\beta+1)\frac{k}{2}-l-\alpha m+2\alpha p-i-2r-1,\beta)\end{matrix}\right.\right] + p_0\sum_{k=0}^{\infty}(-\theta)^k\sum_{j=1}^{\infty}\frac{(-y)^j}{j!}\left(\frac{\lambda_1^\alpha}{\lambda_3^\beta}\right)^{\frac{k+j}{2}}\sum_{l=0}^{\infty}\frac{(-\mathrm{M})^l}{l!}$$
$$\sum_{m=0}^{\infty}\frac{(-\lambda_1^{-\alpha})^m}{m!}\sum_{p=0}^{m}\frac{(\lambda_2^\alpha)^p m!}{p!(m-p)!}\sum_{i=0}^{\infty}(-\mathrm{M})^i\sum_{r=0}^{\infty}(-w^2)^r\,t^{(\beta-\alpha-1)\frac{k+j}{2}+l+\alpha m-2\alpha p+i+2r+1}$$

and, by using Eq. (8), the corresponding stress is found in the form of

$$\tau(y,t)=-\Gamma(b+1)\sum_{k=0}^{\infty}(-\theta)^k\sum_{j=0}^{\infty}\frac{(-y)^j}{j!}\left(\frac{\lambda_1^{\alpha}}{\lambda_3^{\beta}}\right)^{\frac{k+j-1}{2}}\sum_{l=0}^{\infty}\frac{(-\mathrm{M})^l}{l!}\sum_{m=0}^{\infty}\frac{(-\lambda_1^{-\alpha})^m}{m!}\sum_{p=0}^{m}\frac{(\lambda_2^{\alpha})^p m!}{p!(m-p)!}$$
$$t^{(\beta-\alpha-1)\frac{k+j-1}{2}+l+\alpha m-2\alpha p+b-1}\,\mathrm{H}_{3,5}^{1,3}\left[\frac{t^{\beta}}{\lambda_3^{\beta}}\left|\begin{matrix}(1-l+\frac{k+j-1}{2},0),(1-m+\frac{k+j-1}{2},0),(1-\frac{k+j-1}{2},1)\\(0,1),(1+\frac{k+j+1}{2},0),(1+\frac{k+j-1}{2},0),(1-\frac{k+j-1}{2},0),((\alpha-\beta+1)\frac{k+j-1}{2}-l-\alpha m+2\alpha p-b+1,\beta)\end{matrix}\right.\right]-$$
$$p_0\sum_{k=0}^{\infty}(-\theta)^k\sum_{j=0}^{\infty}\frac{(-y)^j}{j!}\left(\frac{\lambda_1^{\alpha}}{\lambda_3^{\beta}}\right)^{\frac{k+j-1}{2}}\sum_{l=0}^{\infty}\frac{(-\mathrm{M})^l}{l!}\sum_{m=0}^{\infty}\frac{(-\lambda_1^{-\alpha})^m}{m!}\sum_{p=0}^{m}\frac{(\lambda_2^{\alpha})^p m!}{p!(m-p)!}\sum_{i=0}^{\infty}(-\mathrm{M})^i\sum_{r=0}^{\infty}(-w^2)^r \tag{30}$$
$$t^{(\beta-\alpha-1)\frac{k+j-1}{2}+l+\alpha m-2\alpha p+i+2r}\,\mathrm{H}_{3,5}^{1,3}\left[\frac{t^{\beta}}{\lambda_3^{\beta}}\left|\begin{matrix}(1-l+\frac{k+j+1}{2},0),(1-m+\frac{k+j-1}{2},0),(1-\frac{k+j-1}{2},1)\\(0,1),(1+\frac{k+j+1}{2},0),(1+\frac{k+j-1}{2},0),(1-\frac{k+j-1}{2},0),((\alpha-\beta+1)\frac{k+j-1}{2}-l-\alpha m+2\alpha p-i-2r,\beta)\end{matrix}\right.\right]$$

Special Cases:

1- If $\theta=0$ then the no- slip condition is obtained. In this special case Eqs. (29) and (30) are simplified into

$$u(y,t)=t^b+\Gamma(b+1)\sum_{j=1}^{\infty}\frac{(-y)^j}{j!}\left(\frac{\lambda_1^{\alpha}}{\lambda_3^{\beta}}\right)^{\frac{j}{2}}\sum_{l=0}^{\infty}\frac{(-\mathrm{M})^l}{l!}\sum_{m=0}^{\infty}\frac{(-\lambda_1^{-\alpha})^m}{m!}\sum_{p=0}^{m}\frac{(\lambda_2^{\alpha})^p m!}{p!(m-p)!}\,t^{(\beta-\alpha-1)\frac{j}{2}+l+\alpha m-2\alpha p+b}$$
$$\mathrm{H}_{3,5}^{1,3}\left[\frac{t^{\beta}}{\lambda_3^{\beta}}\left|\begin{matrix}(1-l+\frac{j}{2},0),(1-m+\frac{j}{2},0),(1-\frac{j}{2},1)\\(0,1),(1+\frac{j}{2},0),(1+\frac{j}{2},0),(1-\frac{j}{2},0),((\alpha-\beta+1)\frac{j}{2}-l-\alpha m+2\alpha p-b,\beta)\end{matrix}\right.\right]+p_0\sum_{j=1}^{\infty}\frac{(-y)^j}{j!}\left(\frac{\lambda_1^{\alpha}}{\lambda_3^{\beta}}\right)^{\frac{j}{2}}\sum_{l=0}^{\infty}\frac{(-\mathrm{M})^l}{l!}\sum_{m=0}^{\infty}\frac{(-\lambda_1^{-\alpha})^m}{m!}$$
$$\sum_{p=0}^{m}\frac{(\lambda_2^{\alpha})^p m!}{p!(m-p)!}\sum_{i=0}^{\infty}(-\mathrm{M})^i\sum_{r=0}^{\infty}(-w^2)^r\,t^{(\beta-\alpha-1)\frac{j}{2}+l+\alpha m-2\alpha p+i+2r+1}\,\mathrm{H}_{3,5}^{1,3}\left[\frac{t^{\beta}}{\lambda_3^{\beta}}\left|\begin{matrix}(1-l+\frac{j}{2},0),(1-m+\frac{j}{2},0),(1-\frac{j}{2},1)\\(0,1),(1+\frac{j}{2},0),(1+\frac{j}{2},0),(1-\frac{j}{2},0),((\alpha-\beta+1)\frac{j}{2}-l-\alpha m+2\alpha p-i-2r-1,\beta)\end{matrix}\right.\right]$$

$$\tau=-\Gamma(b+1)\sum_{j=0}^{\infty}\frac{(-y)^j}{j!}\left(\frac{\lambda_1^{\alpha}}{\lambda_3^{\beta}}\right)^{\frac{j-1}{2}}\sum_{l=0}^{\infty}\frac{(-\mathrm{M})^l}{l!}\sum_{m=0}^{\infty}\frac{(-\lambda_1^{-\alpha})^m}{m!}\sum_{p=0}^{m}\frac{(\lambda_2^{\alpha})^p m!}{p!(m-p)!}$$
$$t^{(\beta-\alpha-1)\frac{j-1}{2}+l+\alpha m-2\alpha p+b-1}\,\mathrm{H}_{3,5}^{1,3}\left[\frac{t^{\beta}}{\lambda_3^{\beta}}\left|\begin{matrix}(1-l+\frac{j-1}{2},0),(1-m+\frac{j-1}{2},0),(1-\frac{j-1}{2},1)\\(0,1),(1+\frac{j+1}{2},0),(1+\frac{j-1}{2},0),(1-\frac{j-1}{2},0),((\alpha-\beta+1)\frac{j-1}{2}-l-\alpha m+2\alpha p-b+1,\beta)\end{matrix}\right.\right]-$$
$$p_0\sum_{j=0}^{\infty}\frac{(-y)^j}{j!}\left(\frac{\lambda_1^{\alpha}}{\lambda_3^{\beta}}\right)^{\frac{j-1}{2}}\sum_{l=0}^{\infty}\frac{(-\mathrm{M})^l}{l!}\sum_{m=0}^{\infty}\frac{(-\lambda_1^{-\alpha})^m}{m!}\sum_{p=0}^{m}\frac{(\lambda_2^{\alpha})^p m!}{p!(m-p)!}\sum_{i=0}^{\infty}(-\mathrm{M})^i\sum_{r=0}^{\infty}(-w^2)^r$$
$$t^{(\beta-\alpha-1)\frac{j-1}{2}+l+\alpha m-2\alpha p+i+2r}\,\mathrm{H}_{3,5}^{1,3}\left[\frac{t^{\beta}}{\lambda_3^{\beta}}\left|\begin{matrix}(1-l+\frac{j+1}{2},0),(1-m+\frac{j-1}{2},0),(1-\frac{j-1}{2},1)\\(0,1),(1+\frac{j+1}{2},0),(1+\frac{j-1}{2},0),(1-\frac{j-1}{2},0),((\alpha-\beta+1)\frac{j-1}{2}-l-\alpha m+2\alpha p-i-2r,\beta)\end{matrix}\right.\right]$$

2- If $\theta\neq 0$ and $\mathrm{M}=0$ then Eqs. (29) and (30) reduce to

$$u(y,t)=t^{b}+\Gamma(b+1)\sum_{k=1}^{\infty}(-\theta)^{k}\left(\frac{\lambda_1^{\alpha}}{\lambda_3^{\beta}}\right)^{\frac{k}{2}}\sum_{m=0}^{\infty}\frac{(-\lambda_1^{-\alpha})^{m}}{m!}\sum_{p=0}^{m}\frac{(\lambda_2^{\alpha})^{p}\,m!}{p!(m-p)!}\,t^{(\beta-\alpha-1)\frac{k}{2}+\alpha m-2\alpha p+b}$$

$$\mathrm{H}_{2,4}^{1,2}\left[\frac{t^{\beta}}{\lambda_3^{\beta}}\middle|\begin{matrix}(1-m+\frac{k}{2},0),(1-\frac{k}{2},1)\\(0,1),(1+\frac{k}{2},0),(1-\frac{k}{2},0),((\alpha-\beta+1)\frac{k}{2}-\alpha m+2\alpha p-b,\beta)\end{matrix}\right]+\Gamma(b+1)\sum_{k=0}^{\infty}(-\theta)^{k}\sum_{j=1}^{\infty}\frac{(-y)^{j}}{j!}\left(\frac{\lambda_1^{\alpha}}{\lambda_3^{\beta}}\right)^{\frac{k+j}{2}}\sum_{m=0}^{\infty}\frac{(-\lambda_1^{-\alpha})^{m}}{m!}$$

$$\sum_{p=0}^{m}\frac{(\lambda_2^{\alpha})^{p}\,m!}{p!(m-p)!}\,t^{(\beta-\alpha-1)\frac{k+j}{2}+\alpha m-2\alpha p+b}\,\mathrm{H}_{2,4}^{1,2}\left[\frac{t^{\beta}}{\lambda_3^{\beta}}\middle|\begin{matrix}(1-m+\frac{k+j}{2},0),(1-\frac{k+j}{2},1)\\(0,1),(1+\frac{k+j}{2},0),(1-\frac{k+j}{2},0),((\alpha-\beta+1)\frac{k+j}{2}-\alpha m+2\alpha p-b,\beta)\end{matrix}\right]+p_0\sum_{k=1}^{\infty}(-\theta)^{k}\left(\frac{\lambda_1^{\alpha}}{\lambda_3^{\beta}}\right)^{\frac{k}{2}}$$

$$\sum_{m=0}^{\infty}\frac{(-\lambda_1^{-\alpha})^{m}}{m!}\sum_{p=0}^{m}\frac{(\lambda_2^{\alpha})^{p}\,m!}{p!(m-p)!}\sum_{r=0}^{\infty}(-w^{2})^{r}\,t^{(\beta-\alpha-1)\frac{k}{2}+\alpha m-2\alpha p+2r+1}\,\mathrm{H}_{2,4}^{1,2}\left[\frac{t^{\beta}}{\lambda_3^{\beta}}\middle|\begin{matrix}(1-m+\frac{k}{2},0),(1-\frac{k}{2},1)\\(0,1),(1+\frac{k}{2},0),(1-\frac{k}{2},0),((\alpha-\beta+1)\frac{k}{2}-\alpha m+2\alpha p-2r-1,\beta)\end{matrix}\right]$$

$$+p_0\sum_{k=0}^{\infty}(-\theta)^{k}\sum_{j=1}^{\infty}\frac{(-y)^{j}}{j!}\left(\frac{\lambda_1^{\alpha}}{\lambda_3^{\beta}}\right)^{\frac{k+j}{2}}\sum_{m=0}^{\infty}\frac{(-\lambda_1^{-\alpha})^{m}}{m!}\sum_{p=0}^{m}\frac{(\lambda_2^{\alpha})^{p}\,m!}{p!(m-p)!}\sum_{r=0}^{\infty}(-w^{2})^{r}\,t^{(\beta-\alpha-1)\frac{k+j}{2}+\alpha m-2\alpha p+2r+1}$$

$$\mathrm{H}_{2,4}^{1,2}\left[\frac{t^{\beta}}{\lambda_3^{\beta}}\middle|\begin{matrix}(1-m+\frac{k+j}{2},0),(1-\frac{k+j}{2},1)\\(0,1),(1+\frac{k+j}{2},0),(1-\frac{k+j}{2},0),((\alpha-\beta+1)\frac{k+j}{2}-\alpha m+2\alpha p-2r-1,\beta)\end{matrix}\right]$$

$$\tau(y,t)=-\Gamma(b+1)\sum_{k=0}^{\infty}(-\theta)^{k}\sum_{j=0}^{\infty}\frac{(-y)^{j}}{j!}\left(\frac{\lambda_1^{\alpha}}{\lambda_3^{\beta}}\right)^{\frac{k+j-1}{2}}\sum_{m=0}^{\infty}\frac{(-\lambda_1^{-\alpha})^{m}}{m!}\sum_{p=0}^{m}\frac{(\lambda_2^{\alpha})^{p}\,m!}{p!(m-p)!}$$

$$t^{(\beta-\alpha-1)\frac{k+j-1}{2}+\alpha m-2\alpha p+b-1}\,\mathrm{H}_{2,4}^{1,2}\left[\frac{t^{\beta}}{\lambda_3^{\beta}}\middle|\begin{matrix}(1-m+\frac{k+j-1}{2},0),(1-\frac{k+j-1}{2},1)\\(0,1),(1+\frac{k+j-1}{2},0),(1-\frac{k+j-1}{2},0),((\alpha-\beta+1)\frac{k+j-1}{2}-\alpha m+2\alpha p-b+1,\beta)\end{matrix}\right]-$$

$$p_0\sum_{k=0}^{\infty}(-\theta)^{k}\sum_{j=0}^{\infty}\frac{(-y)^{j}}{j!}\left(\frac{\lambda_1^{\alpha}}{\lambda_3^{\beta}}\right)^{\frac{k+j-1}{2}}\sum_{m=0}^{\infty}\frac{(-\lambda_1^{-\alpha})^{m}}{m!}\sum_{p=0}^{m}\frac{(\lambda_2^{\alpha})^{p}\,m!}{p!(m-p)!}\sum_{r=0}^{\infty}(-w^{2})^{r}$$

$$t^{(\beta-\alpha-1)\frac{k+j-1}{2}+\alpha m-2\alpha p+2r}\,\mathrm{H}_{2,4}^{1,2}\left[\frac{t^{\beta}}{\lambda_3^{\beta}}\middle|\begin{matrix}(1-m+\frac{k+j-1}{2},0),(1-\frac{k+j-1}{2},1)\\(0,1),(1+\frac{k+j-1}{2},0),(1-\frac{k+j-1}{2},0),((\alpha-\beta+1)\frac{k+j-1}{2}-\alpha m+2\alpha p-2r,\beta)\end{matrix}\right]$$

which correspond to the flow of a generalized Burgers' magnetic field effect.

V. Numerical results and discussion:

In this work, we have discussed the MHD flow of generalized Burger's fluid due to accelerating plate with slip effects. The exact solutions for the velocity field u and the stress τ in terms of the Fox H-function are obtained by using the discrete Laplace transform. Moreover, some figures are plotted to show the behavior of various parameters involved in the expressions of velocity u .

A comparison between non- slip effect (Panel a) and slip effect (Panel b) is also made graphically. Figs. 1-7 are prepared for flow due to constant pressure gradient where as Figs. 8- 14 for flow due to sinusoidal pressure gradient.

Fig. 1 shows the variation of the non- integer fractional parameter α and the slip coefficient θ. The velocity is increasing with the increase of α and θ.

Fig. 2 is depicted to show the changes of the velocity with fractional parameter β . In the case of non- slip condition is fullfield, the influence of β is same as that of α . However, it is observed that as θ increasing there is a variation in velocity value a bout some value of β which is greater than 0.4.

Figs. 3 and 4 provide the graphically illustrations for the effects of relaxation and retardation parameters λ_1 and λ_3 on the velocity fields. The velocity is increasing with the increased the λ_1 and λ_3 for both cases, the slip and no- slip condition.

The effect of λ_2 is illustrated in Fig. 5 which shows that λ_2 has quite the opposite effect to that of λ_1 and λ_3 for both cases ($\theta=0$ & $\theta\neq0$).

Fig. 6 demonstrates the influence of the magnetic field M. It is noticed for both cases $\theta=0$ & $\theta\neq0$, when M < 2 there is increasing in the velocity field, however when M > 2 show an opposite effect on the velocity.

In Fig. 7, the variations of the slip coefficient θ on velocity with the magnetic field parameter. The velocity is decreasing with increase of the magnetic parameter.

Figs. 8- 14 provide the graphically illustrations for the velocity for flow due to sinusoidal pressure gradient. Qualitatively, the observations for sinusoidal pressure gradient flow are similar to that of constant pressure gradient flow. However, the velocity profile in flow of constant and sinusoidal pressure gradient are not similar quantitatively.

Fig. 15 demonstrate the velocity changes with time at given points (y=1 and y=2) for Panel (a) $\theta=0$ and Panel (b) $\theta=0.5$ for two types of flows. Comparison shows that the velocity profile in sinusoidal pressure gradient flow are larger when compared to those of constant pressure gradient flow. The effects of the slip coefficient and magnetic field are the similar on the both flows.

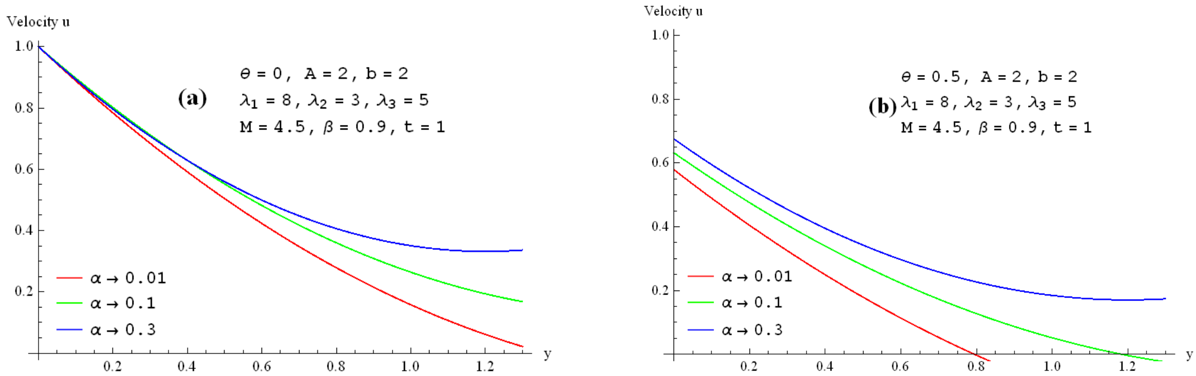

Fig. 1. The velocity for different value of α when keeping other parameters fixed a) $\theta=0$ b) $\theta=0.5$ (Constant p. g.)

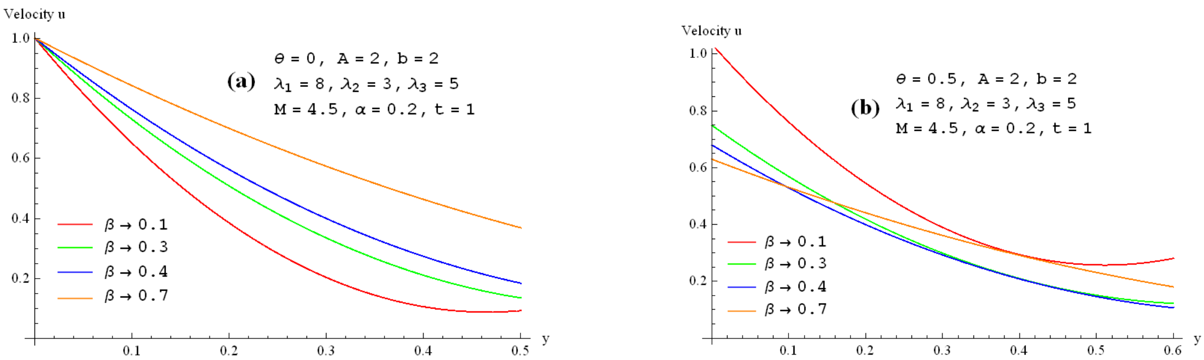

Fig. 2. The velocity for different value of β when keeping other parameters fixed **a)** $\theta=0$ **b)** $\theta=0.5$ (Constant p. g.)

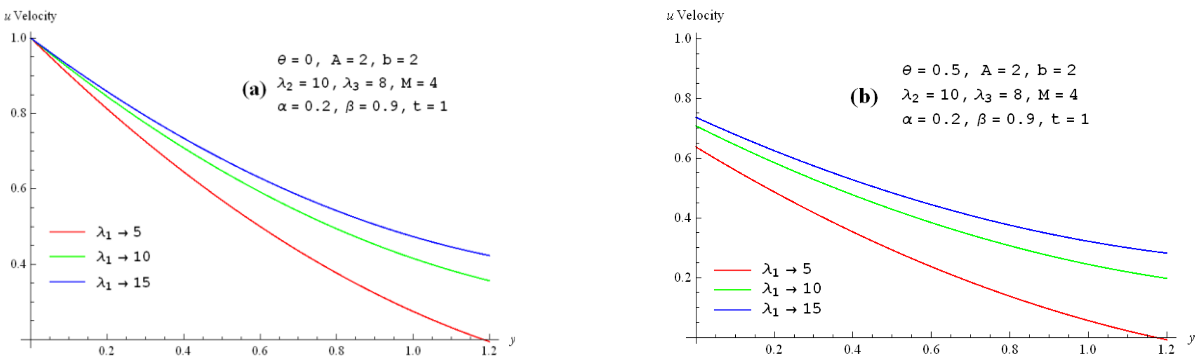

Fig. 3. The velocity for different value of λ_1 when keeping other parameters fixed a) $\theta=0$ b) $\theta=0.5$ (Constant p. g.)

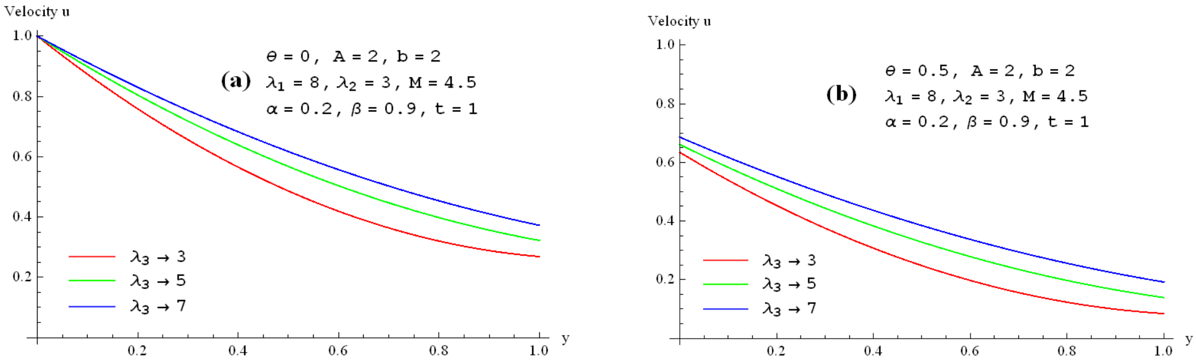

Fig. 4. The velocity for different value of λ_3 when keeping other parameters fixed a) $\theta = 0$ b) $\theta = 0.5$ (Constant p. g.)

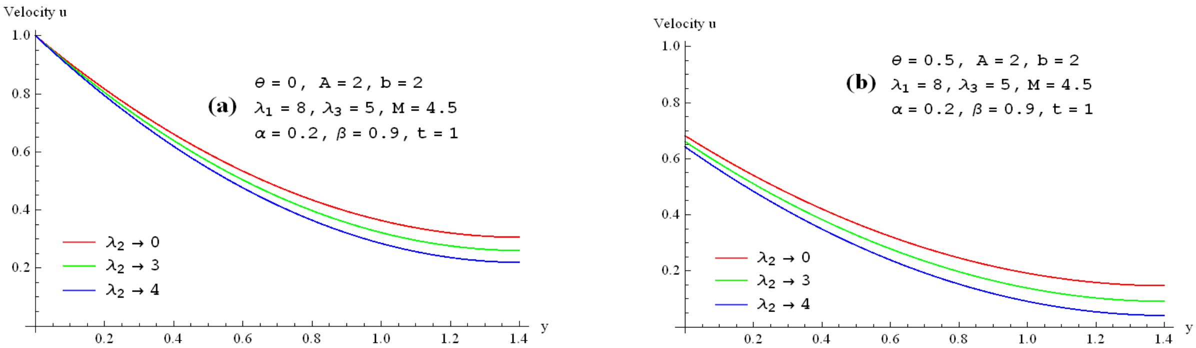

Fig. 5. The velocity for different value of λ_2 when keeping other parameters fixed a) $\theta = 0$ b) $\theta = 0.5$ (Constant p. g.)

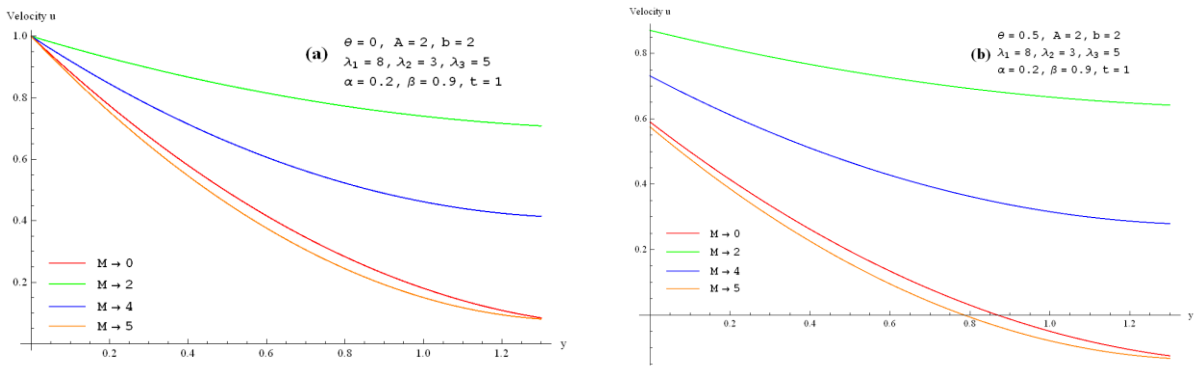

Fig. 6. The velocity for different value of M when keeping other parameters fixed a) $\theta = 0$ b) $\theta = 0.5$ (Constant p. g.)

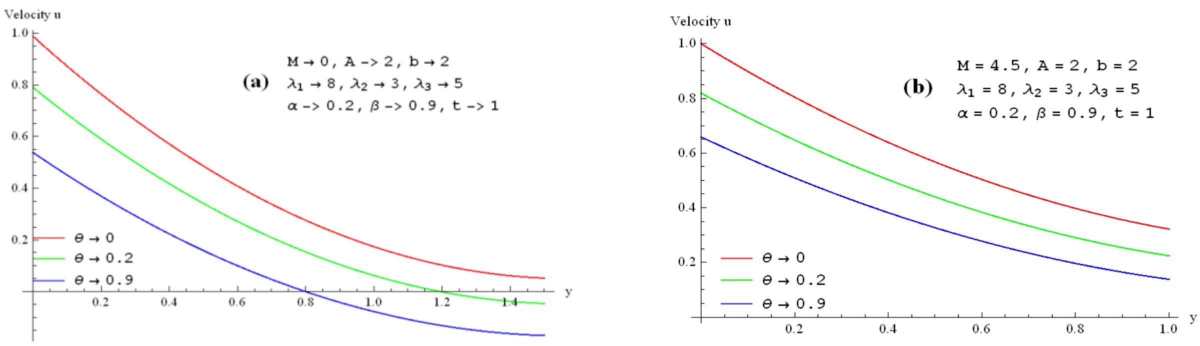

Fig. 7. The velocity for different value of θ when keeping other parameters fixed a) $M = 0$ b) $M = 4.5$ (Constant p. g.)

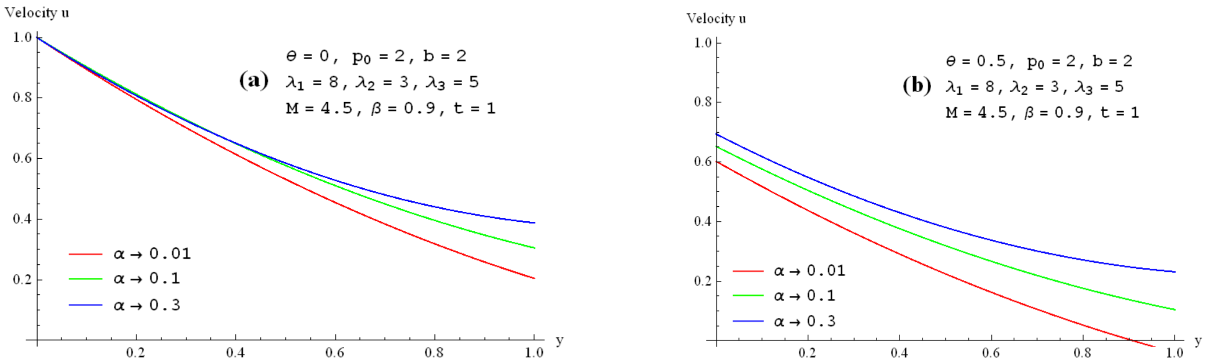

Fig. 8. The velocity for different value of α when keeping other parameters fixed a) $\theta = 0$ b) $\theta = 0.5$ (Sinusoidal p. g.)

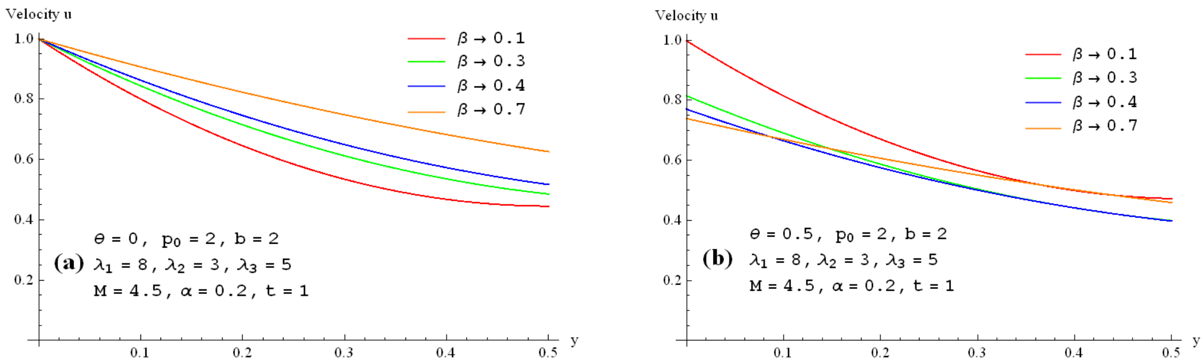

Fig. 9. The velocity for different value of β when keeping other parameters fixed a) $\theta = 0$ b) $\theta = 0.5$ (Sinusoidal p. g.)

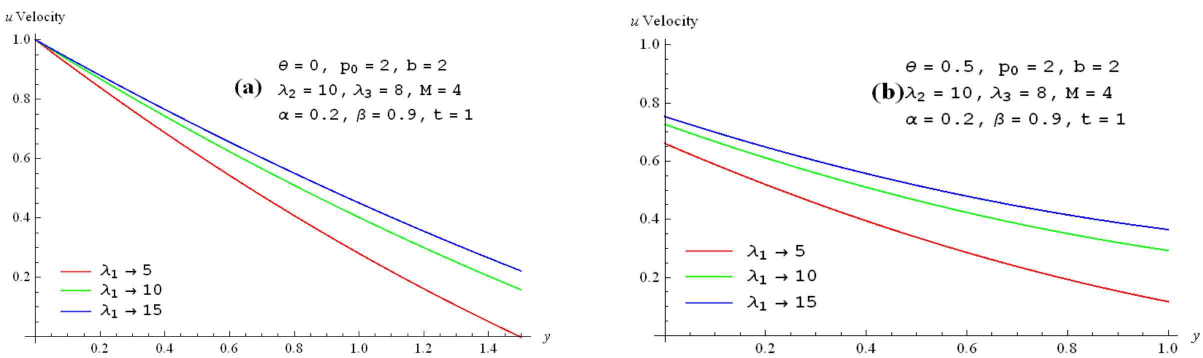

Fig. 10. The velocity for different value of λ_1 when keeping other parameters fixed a) $\theta = 0$ b) $\theta = 0.5$ (Sinusoidal p. g.)

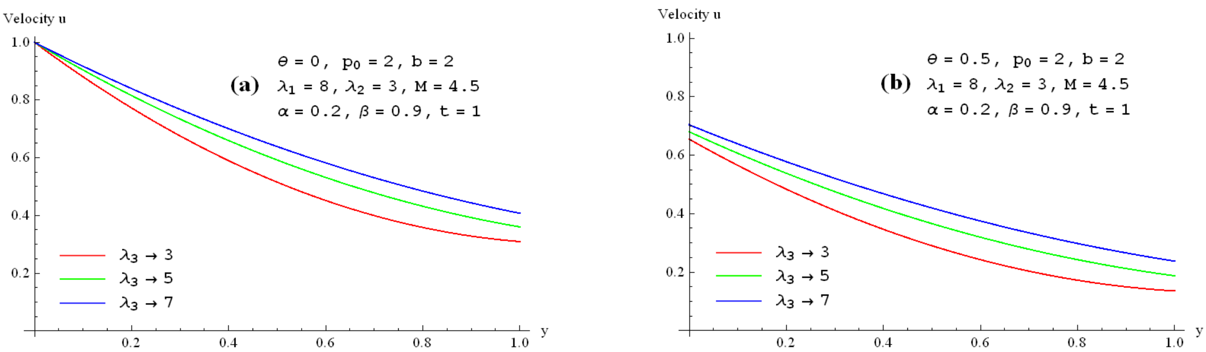

Fig. 11. The velocity for different value of λ_3 when keeping other parameters fixed a) $\theta = 0$ b) $\theta = 0.5$ (Sinusoidal p. g.)

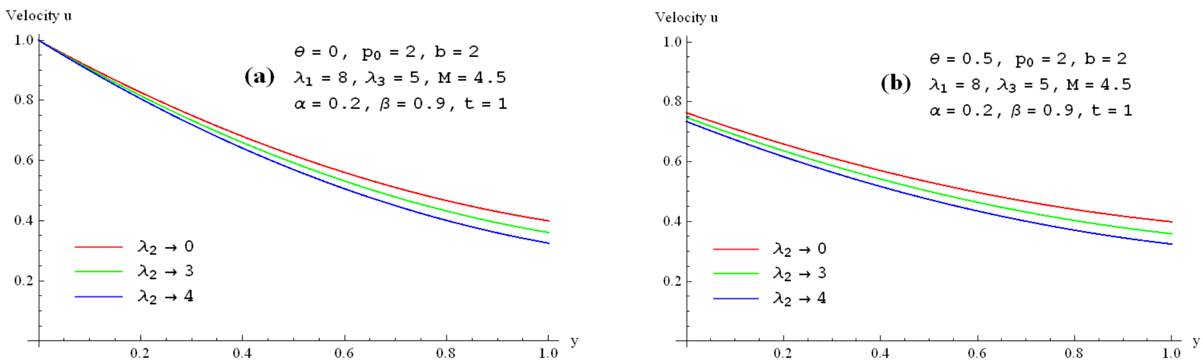

Fig. 12. The velocity for different value of λ_2 when keeping other parameters fixed a) $\theta = 0$ b) $\theta = 0.5$ (Sinusoidal p. g.)

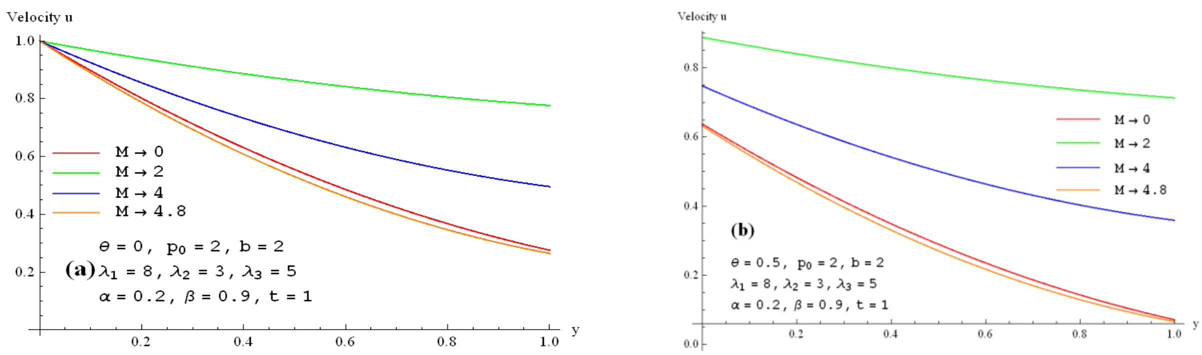

Fig. 13. The velocity for different value of M when keeping other parameters fixed a) $\theta = 0$ b) $\theta = 0.5$ (Sinusoidal p. g.)

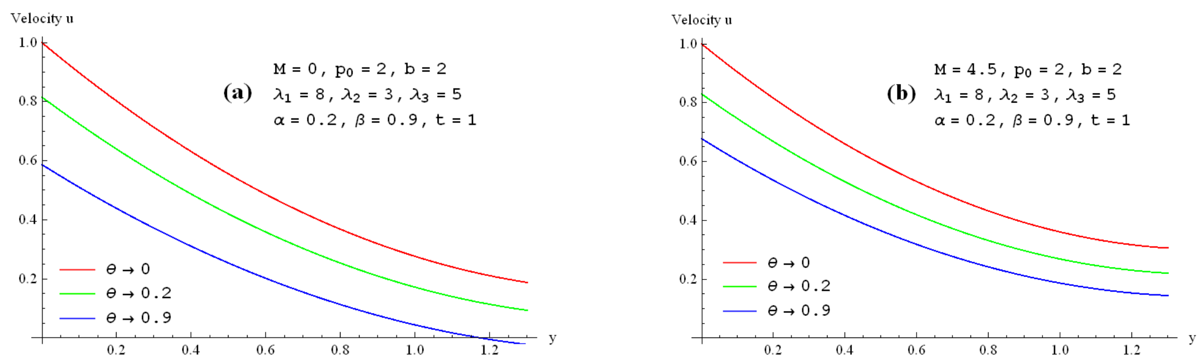

Fig. 14. The velocity for different value of θ when keeping other parameters fixed a) $M = 0$ b) $M = 4.5$ (Sinusoidal p. g.)

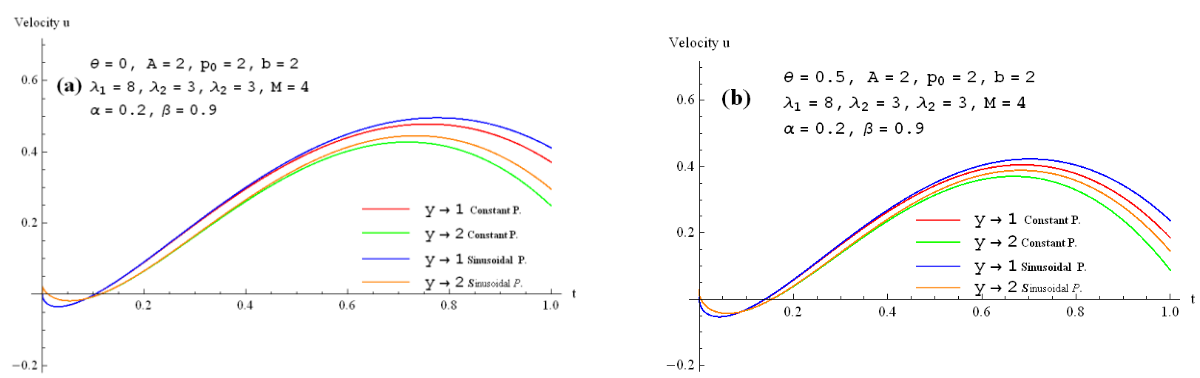

Fig. 15. The velocity for different value of y when keeping other parameters fixed a) $\theta = 0$ b) $\theta = 0.5$ (Constant P. & Sinusoidal P.)

References

[1] A. Ebaid; " *Effects of Mgnetic Field and Wall Slip Conditions on the Peristaltic Transport of a Newtonian Fluid in an Asymmetric Channel*", Phys. Lett. A 372 (2008) 4493- 4499.

[2] A.-R. A. Khaleda and K. Vafaib; " *The Effect of the Slip Condition on Stokes and Couette Flows due to an Oscillating Wall: Exact Solutions*", Int. J. Nonlinear Mech. 39 (2004) 795- 809.

[3] B. H. Tan, I. Jackson and J. D. F. Gerald; " *High- Temperature Viscoelasiticity of Fine- Grained Polycystalline Olivine*", Phys. Chem. Miner. 28 (2001) 641.

[4] C. Derek, D. C. Tretheway and C. D. Meinhart; " *Apparent Fluid Slip at Hydrophibic Microchannel Walls*", Phys. Fluids 14 (2002) 9- 12.

[5] C. Fetecau and Corina Fetecau; " *Decary of Potential Vovtex in Maxwell fluid*", Internet. J. Non- Linear Mech. 38 (2003) 985.

[6] C. Fetecau, T. Hayat and Corina Fetecau; " *Steady- State Solutions for Simple Flows of Generalized Burgers' Fluid*", Int. J. Non- linear Mech. 41 (2006) 880.

[7] I. Podlubny;"*Fractional Differentional Equations*" Academic Press, San Diego, 1999.

[8] J. M. Burgers; " *Mechanical Considerations- Model Systems- Phenomenological Theoies of Relaxation and of Viscosity*", in: J. M. Burgers (Ed.), First Report on Viscosity and Plasticity, Nordemann Publishing Company, New York, 1985.

[9] K. R. Rajagopal and R. K. Bhatnagar; " *Exact Solutions for Some Simple Flows of an Oldroyd- B Fluid*", Acta Mech. 13 (1995) 233.

[10] N. Ali, Q. Hussain, T. Hayat and S. Asghar; " *Slip Effects on the Peristaltic Transport of MHD Fluid with Variable Viscosity*", Phys. Lett. A 372 (2008) 1477- 1489.

[11] P. Ravindran, J. M. Krishnan and K. R. Rajagopal; " *Anote on the Flow of Burgers' Fluid in an Orthogonal Rheometer,* Internat. J. Engrg. Sci. 42 (2004) 1973.

[12] S. Hyder, M. Khan and H. Qi; " *Exact Solutions for a Viscoelastic Fluid with the Generalized Oldroyd- B Model*", Nonlinear Anal.: RWA 10 (2009) 2590- 2599.

[13] T. Hayat, A. M. Siddiqui and S. Asghar; " *Some Simple Flows of an Oldroyd- B Fluid*", Internat. J. Engrg. Sci. 39 (2001) 135.

[14] T. Hayat, C. Fetecau and S. Asghar; " *Some Simple Flows of Burgers' Fluid*", Internat. J. Engrg. Sci. 44 (2006) 1423.

[15] T. Hayat, C. Fetecau and S. Asghar; " *Some Simple Flows of Burgers' Fluid*", Comput. Math. Appl. 52 (2006) 1413.

[16] T. Hayat, M. Hussain and M. Khan; " *Hall Effects on Flows of an Oldriyd- B Fluid Through Porous Medium for Cylindical Geometries*", Comput. Math. Appl. 52 (2006) 333.

[17] T. Hayat, M. Khan and M. Ayub; " *Exact Solutions of Flow Problem of an Oldroyd- B fluid*", Appl. Math. Comput. 151 (2004) 105.

[18] W. R. Peltier, P. We and D. A. Yuen; " *The Viscosities of the Earth Manthe*", in: F. D. Stocey, M. S. Paterson, A. Nicholas (Eds), Auelasticity in the Eearth, American Geophysical Union, Colorado, 1981.

[19] Zheng, L., Liu, Y. and Zheng, X.; " *Slip effect on MHD Flow of a Generalized Oldroyd- B Fluid with Fractional Derivative*", Nonlinear Anal. RWA 13 (2012) 513- 523.